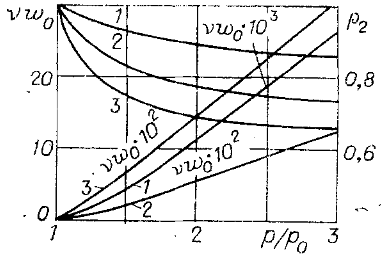

Fig. 3

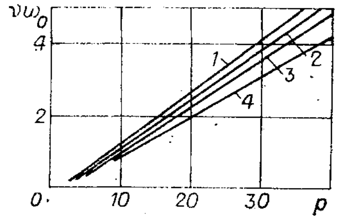

Fig. 4

plate flow. The results of computations of the dependences $\nu w_0 - p$ are presented in Fig. 4 for the values $\xi < \xi_K$ (ξ = 0.1, 0.2, 0.25, 0.3 are the lines 1-4).

One interesting feature of the solutions obtained for viscoplastic deformation problems should be noted in the case of a linear function Φ. The linear dependence of the characteristic rates of deflection on the load (2.5), (2.7) is sufficiently regular for all plates in the presence of one flow mode, however, an analogous dependence in the presence of several zones with moving boundaries is somewhat unexpected. Nevertheless, despite the awkwardness of the analysis, the deviations from the linear dependence are not large in all cases for the known solutions (see [1, 2, 4-6], Figs. 3 and 4), and are remarked only in the domain of load values near the static limit load. For instance, for a circular plate loaded by uniform pressure [4], the deviations from the linear dependence in the whole range of displacement rates do not exceed 1% of the static limit load.

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INVERSE PROBLEM OF MEMBRANE DEFORMATION UNDER CREEP CONDITIONS

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1. Inverse problems of membrane deformation under creep conditions in a given time in a convex surface for minimal energy expenditures occur, for instance, in analyzing technological equipment for pressure treatment of materials in the creep regime [1].

Let us consider a membrane occupying a domain S in the x_1Ox_2 plane that is bounded by the outline γ and is being deformed under the action of external forces q normal to its plane and p_k (k = 1, 2) applied to γ and lying in its plane. The equilibrium equations have the form [2]

$$\frac{\partial\sigma_{kl}}{\partial x_l} = 0 \quad (k = 1, 2), \quad h\sigma_{kl}\frac{\partial^2 w}{\partial x_k \partial x_l} = -q, \tag{1.1}$$

where σ_{kl} (k, l = 1, 2) are stress tensor components, h is the membrane thickness, and w is its deflection. Summation from 1 to 2 is over the repeated subscripts.

The strain tensor components ε_{kl} (k, l = 1, 2) are related to the displacement components u_k (k = 1, 2) in the x_1Ox_2 plane and the deflection w by the following dependences [2]:

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$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right) + \frac{1}{2}\frac{\partial w}{\partial x_k}\frac{\partial w}{\partial x_l} \quad (k,\ l = 1,\ 2). \tag{1.2}$$

We consider that the total strains of the membrane material are comprised of elastic strains subject to Hooke's law and creep strain:

$$\varepsilon_{kl} = a_{klmn}\sigma_{mn} + \varepsilon_{kl}^c \quad (k,\ l = 1,\ 2), \tag{1.3}$$

where the creep strain rates $\eta_{kl} = \overset{\circ}{\varepsilon}{}_{kl}^{c}$ (The dot denotes differentiation with respect to the time t) are potential stress functions

$$\eta_{kl} = \frac{\partial \Phi}{\partial \sigma_{kl}} \quad (k,\ l = 1,\ 2), \tag{1.4}$$

where $\Phi = \Phi(\sigma_{kl})$ is the creep potential that is a convex homogeneous function of degree n + 1 in σ_{kl} (k, l = 1, 2) [3]. The function $\Phi = [1/(n + 1)]W$, where $W = \sigma_{kl}\eta_{kl}$ is the specific power of the energy dissipated during creep which implies the convexity of the functions $W = W(\sigma_{kl})$ and $W = W(\eta_{kl})$ [3], for any two states the following inequality holds [4]

$$W^{(2)} - W^{(1)} \geqslant \frac{n+1}{n}\sigma_{kl}^{(1)}\left(\eta_{kl}^{(2)} - \eta_{kl}^{(1)}\right). \tag{1.5}$$

Let us formulate the inverse problem whose investigation is the purpose of this paper: What external forces $q = q(x_1,\ x_2,\ t)$ and $p_k = p_k(s,\ t)$ (k = 1, 2), where s is the arclength of the contour γ, $0 \leqslant t < t_*$, must be applied to a membrane which is in the natural unstrained state at $t < 0$ such that given values of residual deflections $w_* = w_*(x_1,\ x_2)$ would be obtained at $t = t_*$ after their instantaneous removal and corresponding elastic unloading, and such that the work of these forces expended in deforming the membrane would be minimal? In other words, among all possible loading paths resulting in a given residual surface shape of an initially flat membrane in a given time t_*, the optimal path in the sense of energy expenditure must be selected.

We consider the given surface to be convex, i.e.,

$$\frac{\partial^2 w_*}{\partial x_1^2} < 0, \quad \frac{\partial^2 w_*}{\partial x_1^2}\frac{\partial^2 w_*}{\partial x_2^2} - \left(\frac{\partial^2 w_*}{\partial x_1 \partial x_2}\right)^2 > 0, \tag{1.6}$$

and also that $w_* = u_k^* = 0$ (k = 1, 2) on γ, where u_k^* are residual displacements in the plane of of the membrane.

It can be shown that the creep strain components are compatible for $t = t_*$, i.e., relationships of the type of (1.2) are expressible in terms of u_k^* (k = 1, 2) and w_*. In fact, after unloading the field of residual stresses σ_{kl}^* and residual deflection w_* should satisfy a system of equations of the form (1.1) for $t = t_*$, in which we should set q = 0 [5]. If the residual stress function $F_* = F_*(x_1,\ x_2)$ is introduced in the usual manner such that the first two equations in (1.1) are satisfied identically, then the third equation in (1.1) will take the form

$$\frac{\partial^2 w_*}{\partial x_2^2}\frac{\partial^2 F_*}{\partial x_1^2} - 2\frac{\partial^2 w_*}{\partial x_1 \partial x_2}\frac{\partial^2 F_*}{\partial x_1 \partial x_2} + \frac{\partial^2 w_*}{\partial x_1^2}\frac{\partial^2 F_*}{\partial x_2^2} = 0. \tag{1.7}$$

Since $p_k^* = 0$ (k = 1, 2) on γ for $t = t_*$, then the boundary conditions for F_* can be reduced to the form [6] $\partial F_*/\partial x_k = 0$ (k = 1, 2) or $F_* = \partial F_*/\partial n = 0$ on γ. By virtue of (1.6), the equation (1.7) for the function $F_* = F_*(x_1,\ x_2)$ is elliptic [7] and on the basis of the boundary conditions mentioned has the unique solution $F_* = 0$, from which $\sigma_{kl}^* = 0$ (k, l = 1, 2).

There results from (1.2) and (1.3)

$$\varepsilon_{kl}^c(t_*) = \frac{1}{2}\left(\frac{\partial u_k^*}{\partial x_l} + \frac{\partial u_l^*}{\partial x_k}\right) + \frac{1}{2}\frac{\partial w_*}{\partial x_k}\frac{\partial w_*}{\partial x_l} \quad (k,\ l = 1,2). \tag{1.8}$$

Let us now calculate the work A expended by the forces q and p_k (k = 1, 2) in membrane deformation under the assumption that w = 0 on γ during the whole process, i.e., for $0 \leqslant t \leqslant t_*$. We have

$$A = I_1 + I_2, \quad I_1 = \int\limits_S \int\limits_0^{w_*} q\,dw\,dx_1 dx_2, \quad I_2 = \int\limits_\gamma \int\limits_0^{u_k^*} p_k\,du_k\,ds.$$

By virtue of (1.1) and the known Green's formula that reduces integration over the domain S to integration over the contour γ, we can obtain

$$I_1 = -h\int_S\int_0^{w_*}\frac{\partial}{\partial x_l}\left(\sigma_{kl}\frac{\partial w}{\partial x_k}dw\right)dx_1dx_2 + I_3 = -h\int_\gamma\int_0^{w_*}\sigma_{kl}\frac{\partial w}{\partial x_k}n_l dw ds + I_3,$$

$$I_3 = h\int_S\int_0^{w_*}\sigma_{kl}\frac{\partial w}{\partial x_k}d\left(\frac{\partial w}{\partial x_l}\right)dx_1dx_2,$$

where n_k (k = 1, 2) are components of the external unit normal vector to γ. Because of the boundary conditions for w the first integral in the last equality vanishes; therefore, $A = I_3 + I_2$.

It is easy to see that the quantity A equals the work of the stress σ_{kl} on the strain ε_{kl} in the whole membrane volume, i.e., $A = h\int_S\int_0^{\varepsilon^*_{kl}}\sigma_{kl}d\varepsilon_{kl}dx_1dx_2$, from which, by virtue of (1.3) and the equalities $\sigma^*_{kl} = 0$ (k, l = 1, 2), we obtain $A = h\int_S\int_0^{t_*}W dt dx_1dx_2$, $W = \sigma_{kl}\eta_{kl}$.

Let us prove the following assertion: The optimal loading path (in the above-mentioned sense) is that for which the stress components at each point of the membrane are independent of the time. Such a stress field, if it exists, is uniquely defined.

We assume that such a path exists; all the quantities referring to it will be denoted with the subscript 0. Then for any other loading, assuring the given residual deflection $w_* = w_*(x_1, x_2)$ after unloading at $t = t_*$, we have

$$A - A_0 = h\int_S\int_0^{t_*}(W - W_0)\,dt dx_1dx_2 \geqslant h\frac{n+1}{n}\int_S\int_0^{t_*}\sigma_{kl_0}(\eta_{kl} - \eta_{kl_0})\,dt dx_1dx_2$$

$$= h\frac{n+1}{n}\int_S\sigma_{kl_0}\Delta\varepsilon^c_{kl}(t_*)\,dx_1dx_2 = h\frac{n+1}{n}\int_S\sigma_{kl_0}\frac{1}{2}\left(\frac{\partial\Delta u^*_k}{\partial x_l} + \frac{\partial\Delta u^*_l}{\partial x_k}\right)dx_1dx_2 \tag{1.9}$$

$$= h\frac{n+1}{n}\int_S\frac{\partial}{\partial x_l}\left(\sigma_{kl_0}\Delta u^*_k\right)dx_1dx_2 = \frac{n+1}{n}\int_\gamma p_{k0}\Delta u^*_k ds = 0. \tag{1.9}$$

In (1.9) we used the inequality (1.5), the condition of independence of σ_{kl_0} from t, the relationships (1.8) in which $w_* = w_*(x_1, x_2)$ is the given function, i.e., $\Delta\left(\frac{\partial w_*}{\partial x_k}\frac{\partial w_*}{\partial x_l}\right) = 0$ (k, l = 1, 2), the Green's formula, and the boundary conditions for the residual displacements u^*_k. The symbol Δ denotes the difference between appropriate quantities referring to the loading paths under consideration. Therefore, $A_0 \leqslant A$, which proves the first part of the assertion.

The proof of the second part is analogous to the proof of the uniqueness theorem for steady creep problems [8]. Indeed, $\varepsilon^c_{kl}(t_*) = \eta_{kl}t_*$ (k, l = 1, 2) follows from (1.4) and we obtain from (1.5) by interchanging the roles of the first and second states and combining the inequality obtained with (1.5)

$$\Delta\sigma_{kl}\Delta\eta_{kl} \geqslant 0,\quad \Delta\sigma_{kl} = \sigma^{(2)}_{kl} - \sigma^{(1)}_{kl},\quad \Delta\eta_{kl} = \eta^{(2)}_{kl} - \eta^{(1)}_{kl}. \tag{1.10}$$

The inequality (1.10) expresses the known Drucker postulate for viscous strain [8]. Assuming the existence of two solutions corresponding to the very same residual deflection w_* and satisfying the zero boundary conditions for u_k (k = 1, 2) with time-independent stress fields and performing calculations analogous to those used in (1.9), we find

$$h\int_S\Delta\sigma_{kl}\Delta\varepsilon^c_{kl}(t_*)\,dx_1dx_2 = ht_*\int_S\Delta\sigma_{kl}\Delta\eta_{kl}dx_1dx_2 = 0,$$

which is possible, by virtue of (1.10), if and only if $\Delta\sigma_{kl} = 0$ (k, l = 1, 2) in the whole volume of the membrane since the expression $\Delta\sigma_{kl}\Delta\eta_{kl}$ is a positive definite quadratic form in $\Delta\sigma_{kl}$ (k, l = 1, 2) [8]. The assertion is proved.

The contour loads for a known stress field σ_{kl} are determined by the dependences $p_k = h\sigma_{kl}n_l$ (k = 1, 2) on γ. It is seen from (1.1) that $w = w(x_1, x_2, t)$ $(0 \leqslant t < t_*)$ must be determined to find the transverse loads $q = q(x_1, x_2, t)$. Eliminating the quantity u_k (k = 1, 2) from (1.2) and taking into account that the creep strain rate η_{kl} (k, l = 1, 2) is independent of t, i.e., $\varepsilon_{kl}^c(t) = \frac{t}{t_*}\varepsilon_{kl}^c(t_*)$, by using the relationships (1.3) and (1.8) we obtain the following equality at any time t $(0 \leqslant t < t_*)$

$$\left(\frac{\partial^2 w}{\partial x_1 \partial x_2}\right)^2 - \frac{\partial^2 w}{\partial x_1^2}\frac{\partial^2 w}{\partial x_2^2} = \frac{1}{E}\Delta\Delta F + \frac{t}{t_*}\left[\left(\frac{\partial^2 w_*}{\partial x_1 \partial x_2}\right)^2 - \frac{\partial^2 w_*}{\partial x_1^2}\frac{\partial^2 w_*}{\partial x_2^2}\right], \tag{1.11}$$

which is the strain compatibility equation [2] for this case. It was assumed for simplicity in the derivation of (1.11) that the membrane material is isotropic, where E is Young's modulus, F is a stress function corresponding to the field σ_{kl}, and $\Delta\Delta$ is the biharmonic operator.

The relationship (1.11) is a Monge—Ampere equation in the unknown deflection w. Its Dirichlet problem with the above-mentioned boundary condition w = 0 on γ has a unique solution, at least for a negative right side (The other solution differs just by a sign) [7].

If the time t_* is sufficiently large, then the components of the stress σ_{kl} will evidently be small quantities; consequently, the elastic strains (constant in time) can be neglected in comparison with the developed creep strains, i.e., the steady creep scheme can be used [3]. Then the first term in the right side of (1.11) can be omitted, and the ellipticity condition for this equation can be satisfied by taking account of (1.6), implying the uniqueness (to the accuracy of a sign) of its solution [7]. In this case, evidently $w = \sqrt{t/t_*}w_*$.

2. Let us consider a rectangular membrane with the sides $2a$ and 2b, $a/b = \varepsilon < 1$. Let us select the origin at the center of the membrane, and let us denote the axes by x and y so that the domain S is determined by the inequalities $|x| \leqslant a$, $|y| \leqslant b$. Let us determine the optimal stress field σ_{kl} (constant in time) of which we spoke above, for this case. As the potential Φ we take the standard [3] from (1.4): $\Phi = \frac{B}{n+1}\sigma_i^{n+1}$, where $\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\sigma_{xy}^2}$ is the intensity of the stress B, n are constants, n > 1.

Let us introduce the dimensionless coordinates $\tilde{x} = x/a$, $\tilde{y} = y/b$, the displacements $\tilde{u} = u/a$, $\tilde{v} = v/a$, and the deflection $\tilde{w} = w/a$ in the xOy plane, later discarding the tilde symbol ~ above the dimensionless quantities so that the domain S will be defined by the inequalities $|x| \leqslant 1$, $|y| \leqslant 1$.

To solve the problem, we apply the method of perturbations [9] by selecting the quantity as small parameter. We shall assume that the given residual deflection $w_* = w_*(x, y)$ depends only on the dimensionless coordinates and does not contain the parameter ε, where $w_*(\pm 1, y) = w_*(x, \pm 1) = 0$. For simplicity we consider that w_* is an even function in both the variables, i.e., $w_*(x, y) = w_*(-x, y) = w_*(x, -y)$. The residual displacements u_* and v_* satisfy the zero boundary conditions, i.e., $u_* = v_* = 0$ for x = ±1 and y = ±1. We later omit the asterisk subscript * on the quantities w_*, u_*, and v_*.

Under the assumptions made for the creep strain components for $t = t_*$, we obtain from (1.4) and (1.8)

$$\begin{aligned}
\varepsilon_x^c(t_*) &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = Bt_*\sigma_i^{n-1}\left(\sigma_x - \frac{1}{2}\sigma_y\right),\\
\varepsilon_y^c(t_*) &= \varepsilon\frac{\partial v}{\partial y} + \frac{\varepsilon^2}{2}\left(\frac{\partial w}{\partial y}\right)^2 = Bt_*\sigma_i^{n-1}\left(\sigma_y - \frac{1}{2}\sigma_x\right),\\
\varepsilon_{xy}^c(t_*) &= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \varepsilon\frac{\partial u}{\partial y}\right) + \frac{\varepsilon}{2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = Bt_*\sigma_i^{n-1}\frac{3}{2}\sigma_{xy}.
\end{aligned} \tag{2.1}$$

The first two equilibrium equations in (1.1) take the form

$$\partial\sigma_x/\partial x + \varepsilon\partial\sigma_{xy}/\partial y = 0, \quad \partial\sigma_{xy}/\partial x + \varepsilon\partial\sigma_y/\partial y = 0. \tag{2.2}$$

Using the usual method [9], we represent the magnitudes of the displacements and stresses in the form of power series in ε and then isolating terms with identical powers in (2.1) and (2.2). Thus, we have for the zeroth approximation

$$\frac{\partial u_0}{\partial x} = Bt_*\sigma_{i0}^{n-1}\left(\sigma_{x0} - \frac{1}{2}\sigma_{y0}\right) - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,$$

$$0 = Bt_* \sigma_{i0}^{n-1}\left(\sigma_{y0} - \frac{1}{2}\sigma_{x0}\right),$$
$$\frac{\partial v_0}{\partial x} = 3Bt_* \sigma_{i0}^{n-1}\sigma_{xy0}, \quad \frac{\partial \sigma_{x0}}{\partial x} = \frac{\partial \sigma_{xy0}}{\partial x} = 0. \tag{2.3}$$

We use the boundary conditions for the variable x, i.e., $u_0|_{x=\pm 1} = v_0|_{x=\pm 1} = 0$, to solve the system (2.3). Consequently, it is not difficult to obtain

$$\sigma_{x0} = \sigma_0, \quad \sigma_{y0} = \frac{1}{2}\sigma_0, \quad \sigma_{xy0} = 0, \quad \sigma_{i0} = \frac{\sqrt{3}}{2}\sigma_0,$$
$$\sigma_0 = \left[\frac{1}{2Bt_*}\left(\frac{2}{\sqrt{3}}\right)^{n+1}\int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx\right]^{\frac{1}{n}}, \tag{2.4}$$
$$u_0 = \frac{x}{2}\int_0^1 \left(\frac{\partial w}{\partial x}\right)^2 dx - \frac{1}{2}\int_0^x \left(\frac{\partial w}{\partial x}\right)^2 dx, \quad v_0 = 0.$$

Evenness of the function w = w(x, y) is used in (2.4). Because w(x, ±1) = 0, we obtain $\partial w/\partial x|_{y=\pm 1} = 0$, from which $u_0|_{y=\pm 1} = 0$. Therefore, the solution (2.4) of the system (2.3) for the zeroth approximation satisfies all the boundary conditions.

We have the system of first approximation conditions from (2.1) and (2.2):

$$\frac{\partial u_1}{\partial x} = Bt_*\left(\frac{\sqrt{3}}{2}\sigma_0\right)^{n-1}\left(\frac{3n+1}{4}\sigma_{x1} - \frac{1}{2}\sigma_{y1}\right),$$
$$\frac{\partial v_0}{\partial y} = Bt_*\left(\frac{\sqrt{3}}{2}\sigma_0\right)^{n-1}\left(\sigma_{y1} - \frac{1}{2}\sigma_{x1}\right),$$
$$\frac{\partial v_1}{\partial x} + \frac{\partial u_0}{\partial y} = 3Bt_*\left(\frac{\sqrt{3}}{2}\sigma_0\right)^{n-1}\sigma_{xy1} - \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \tag{2.5}$$
$$\frac{\partial \sigma_{x1}}{\partial x} + \frac{\partial \sigma_{xy0}}{\partial y} = 0, \quad \frac{\partial \sigma_{xy1}}{\partial x} + \frac{\partial \sigma_{y0}}{\partial y} = 0.$$

The solution of the system (2.5) with the boundary conditions $u_1|_{x=\pm 1} = v_1|_{x=\pm 1} = 0$: is

$$\sigma_{x1} = \sigma_{y1} = 0, \quad \sigma_{xy1} = -\frac{x}{2}\frac{\partial \sigma_0}{\partial y}, \quad u_1 = 0,$$
$$v_1 = \left(\frac{1}{n} + \frac{1}{2}\right)(1 - x^2)\int_0^1 \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x \partial y}dx + \int_x^1\left(\int_0^x \frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}dx\right)dx. \tag{2.6}$$

The equalities (2.4) were used in (2.6). It is seen that $v_1|_{y=\pm 1} = 0$, i.e., all the boundary conditions are satisfied even for the first approximation.

We obtain the system of second approximation equations from (2.1) and (2.2):

$$\frac{\partial u_2}{\partial x} = Bt_*\left(\frac{\sqrt{3}}{2}\right)^{n-1}\sigma_0^{n-2}\left\{\sigma_0\left(\frac{3n+1}{4}\sigma_{x2} - \frac{1}{2}\sigma_{y2}\right) + \frac{n-1}{2}\left[\frac{3(n+1)}{4}\sigma_{x1}^2 + \sigma_{y1}^2 - 2\sigma_{x1}\sigma_{y1} + 3\sigma_{xy1}^2\right]\right\},$$
$$\frac{\partial v_1}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = Bt_*\left(\frac{\sqrt{3}}{2}\right)^{n-1}\sigma_0^{n-2}\left\{\sigma_0\left(\sigma_{y2} - \frac{1}{2}\sigma_{x2}\right) + (n-1)\sigma_{x1}\left(\sigma_{y1} - \frac{1}{2}\sigma_{x1}\right)\right\},$$
$$\frac{\partial v_2}{\partial x} + \frac{\partial u_1}{\partial y} = 3Bt_*\left(\frac{\sqrt{3}}{2}\right)^{n-1}\sigma_0^{n-2}\{\sigma_0\sigma_{xy2} + (n-1)\sigma_{x1}\sigma_{xy1}\}, \tag{2.7}$$
$$\partial\sigma_{x2}/\partial x + \partial\sigma_{xy1}/\partial y = 0, \quad \partial\sigma_{xy2}/\partial x + \partial\sigma_{y1}/\partial y = 0.$$

The solution of the system (2.7) satisfying the boundary conditions $u_2|_{x=\pm 1} = v_2|_{x=\pm 1} = 0$ is

$$\sigma_{x2} = \frac{x^2}{4}\frac{\partial^2 \sigma_0}{\partial y^2} + \frac{1}{n\sigma_0}\left[\frac{1}{2Bt_*}\left(\frac{2}{\sqrt{3}}\right)^{n+1}\frac{1}{\sigma_0^{n-2}}\int_0^1 \Phi_1 dx - \Phi_2\right],$$
$$\sigma_{y2} = \frac{1}{2}\sigma_{x2} + \frac{1}{Bt_*}\left(\frac{2}{\sqrt{3}}\right)^{n-1}\frac{\Phi_1}{\sigma_0^{n-1}}, \quad \sigma_{xy2} = 0,$$
$$u_2 = \frac{x}{2}\int_0^1 \Phi_1 dx - \frac{1}{2}\int_0^x \Phi_1 dx + Bt_*\left(\frac{\sqrt{3}}{2}\right)^{n+1}\sigma_0^{n-2}\Phi_2(x^3 - x), \quad v_2 = 0,$$

$$\Phi_1 = \frac{\partial v_1}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \Phi_2 = \frac{n\sigma_0}{12}\frac{\partial^2 \sigma_0}{\partial y^2} + \frac{n-1}{6}\left(\frac{\partial \sigma_0}{\partial y}\right)^2. \tag{2.8}$$

The following remark must be made with respect to the solution obtained above for the first approximation. As is seen from (2.4), $\sigma_0|_{y=\pm 1} = 0$ which can result in infinite stresses σ_{xy1}, σ_{x2}, and σ_{y2} for $y = \pm 1$. In turn this imposed definite constraints on the applicability of (2.6) and (2.8).

For instance, let $w = \alpha(1 - y^2)^p Q(x)$, where α, p are constants and $Q(\pm 1) = 0$. For the quantities $\partial^k \sigma_0/\partial y^k$ ($k = 1, 2, \ldots$) that will be in the expressions for the higher order approximations of the stresses to be finite, it is necessary that $2p/n$ be a natural number. Thus, $p \geqslant n$ follows from the condition of boundedness of the stresses σ_{xy1}, σ_{x2}, and σ_{y2} for $y = \pm 1$. In this case $u_2(x, \pm 1) = 0$, i.e., all the boundary conditions are satisfied for the second approximation also.

Let us consider an example: $n = 3$, $w = \alpha(1 - y^2)^3(1 - x^2)$. We find from (2.4), (2.6), and (2.8)

$$\sigma_x = \beta\left\{(1 - y^2)^2 + \varepsilon^2\left[x^2(3y^2 - 1) + \frac{137y^2 + 1}{45}\right]\right\}, \quad \beta = \frac{4}{3}\left(\frac{\alpha^2}{2Bt_*}\right)^{\frac{1}{3}},$$

$$\sigma_y = \beta\left\{\frac{(1 - y^2)^2}{2} + \varepsilon^2\left[\frac{9}{8}x^4(7y^2 + 1) - \frac{1}{4}x^2(189y^2 - 1) + \frac{15263y^2 - 671}{360}\right]\right\},$$

$$\sigma_{xy} = 2\beta\varepsilon xy(1 - y^2),$$

$$u = \frac{2}{3}\alpha^2(1 - y^2)^4(x - x^3)\left\{(1 - y^2)^2 + \frac{\varepsilon^2}{60}[9x^2(7y^2 + 1) - 1087y^2 + 79]\right\},$$

$$v = -\alpha^2\varepsilon(1 - x^2)\left(x^2 + \frac{5}{3}\right)y(1 - y^2)^5.$$

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